

UNIVERSITY OF SASKATCHEWAN
GE 124.3 – Engineering Mechanics I
FINAL EXAMINATION

TIME: 3 HOURS

December 17, 2003

Only calculators, pens, pencils, and drawing aids are allowed for the exam.

Candidates are to answer all questions. All questions are of equal value.

You are to show your solution in the space below the question.

The reverse side of the page may be used if required.

NAME:

SOLUTIONS

(First Name)

(Last Name)

Section Number (Day/time): _____

Student Number: _____

Examination Room: _____

Marks

1. _____ /10

2. _____ /10

3. _____ /10

4. _____ /10

5. _____ /10

6. _____ /10

7. _____ /10

8. _____ /10

TOTAL: _____

NOTE:

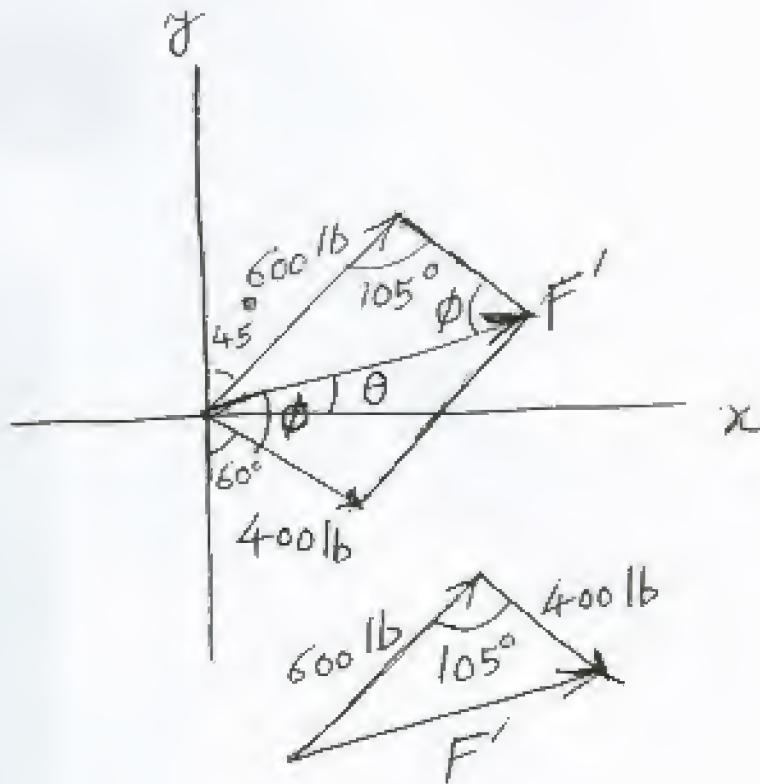
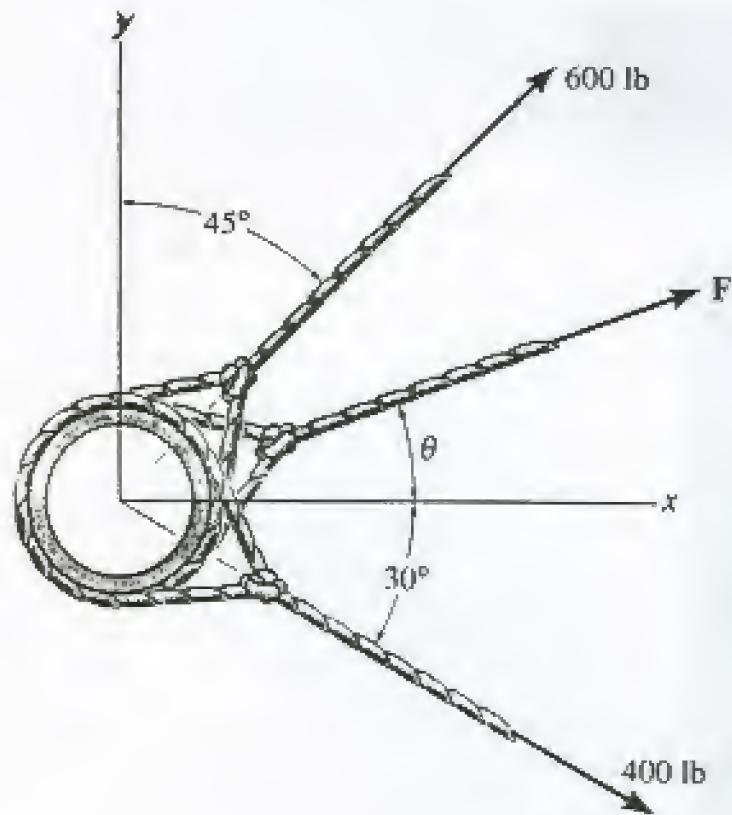
- make sure you have 8 problems in the exam
- use the space below each problem for your solution
- use the back of previous sheet if more room is required for your solution
- each question is worth the indicated marks
- please place your name at the top of each sheet
- a list of formulas is printed on the last sheet of this exam.

EXAM LOCATIONS:

Section 01 Education Gym

1. Three cables pull on the pipe such that they create a resultant force of 900 lb magnitude. All forces lie in the x-y plane. Two of the cables are subjected to known forces of 600 lb and 400 lb as shown.

Determine the direction θ of the third cable so that the magnitude of force \mathbf{F} in this cable is *minimum*. Find the magnitude of \mathbf{F} . (Hint: First find the resultant of the two known forces.)



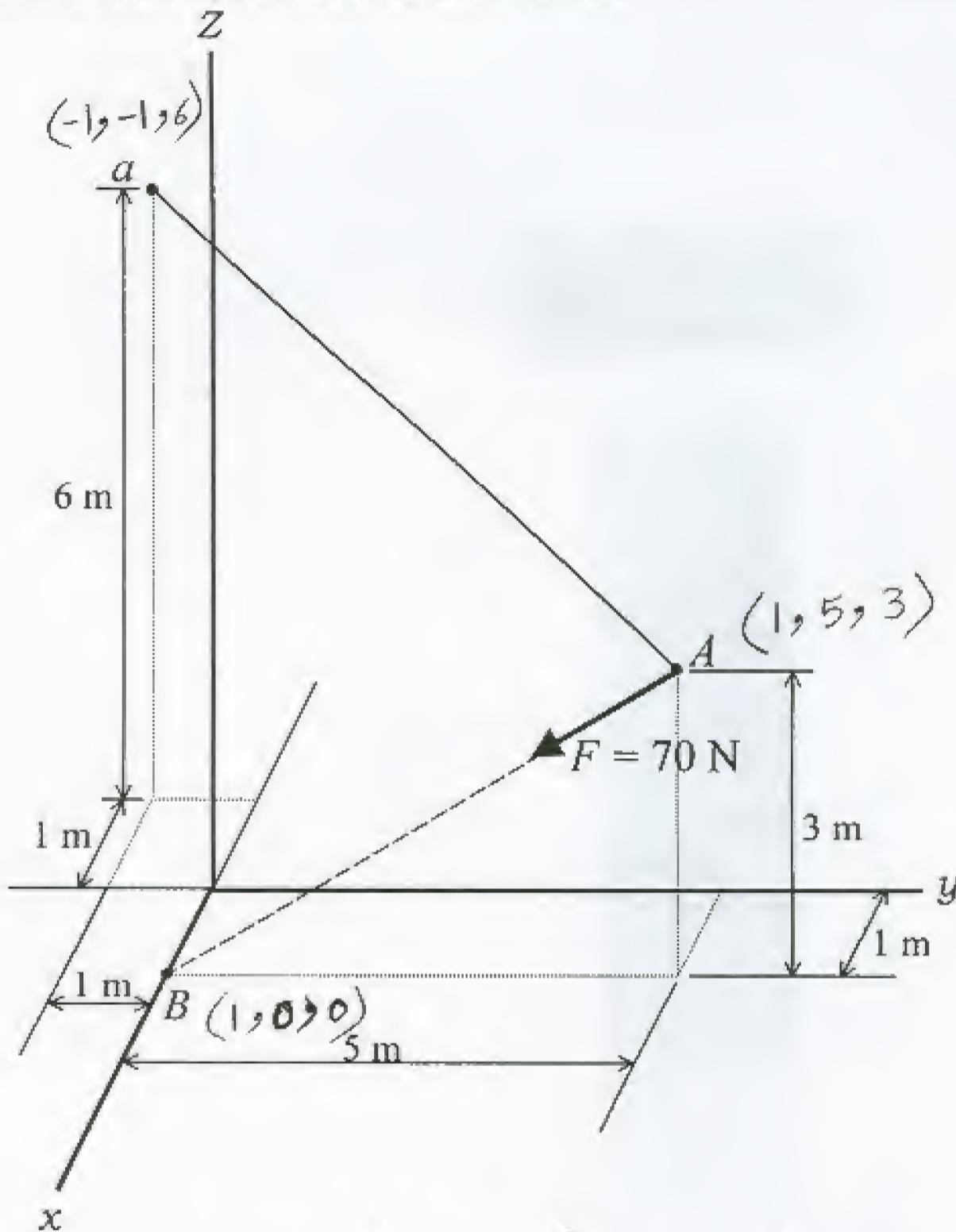
$$F' = \sqrt{(600)^2 + (400)^2 - 2(600)(400)\cos 105^\circ} = 302.64 \text{ lb}$$

$$F = 900 - 302.64 = 97.36 \text{ lb} \rightarrow F' = 97.4 \text{ lb}$$

$$\frac{\sin \phi}{600} = \frac{\sin 105^\circ}{302.64} \Rightarrow \phi = 46.22^\circ$$

$$\theta = 46.22^\circ - 30^\circ = 16.22^\circ \rightarrow \theta = 16.22^\circ$$

2. Determine the magnitudes of the projected components of the 70N force F_{AB} lying parallel to line Aa and lying perpendicular to the line Aa .



$$\vec{F} = 70 \text{ N} \cdot \frac{(1-1)\hat{i} + (0-5)\hat{j} + (0-3)\hat{k}}{\sqrt{0^2 + (-5)^2 + (-3)^2}} = \frac{70}{\sqrt{34}} \left\{ -5\hat{j} - 3\hat{k} \right\} = \left\{ -60.02\hat{j} - 36.01\hat{k} \right\} \text{ N}$$

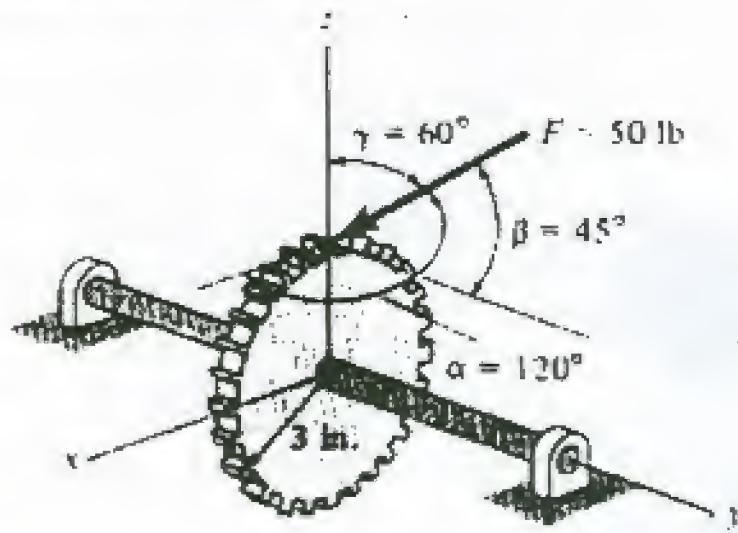
$$\hat{u}_{Aa} = \frac{(-1-1)\hat{i} + (-1-5)\hat{j} + (6-3)\hat{k}}{\sqrt{(-2)^2 + (-6)^2 + (3)^2}} = \frac{-2\hat{i} - 6\hat{j} + 3\hat{k}}{7}$$

$$\vec{F}_{||} = \vec{F} \cdot \hat{u}_{Aa} = (-60.02) \left(-\frac{6}{7} \right) + (-36.01) \left(\frac{3}{7} \right) = 36.01 \text{ N}$$

$$\boxed{\vec{F}_{||} = 36.0 \text{ N}}$$

$$F_{\perp} = \sqrt{F^2 - F_{||}^2} = \sqrt{70^2 - (36.01)^2} = 60.0 \text{ N} \Rightarrow \boxed{F_{\perp} = 60.0 \text{ N}}$$

3. The 50-lb force acts on the 3-inch radius gear in the direction shown. The force direction cosine angles are $\alpha = 120^\circ$, $\beta = 45^\circ$ and $\gamma = 60^\circ$. Determine the moment of this force about the y-axis of the shaft.



$$M_y = \hat{j} \cdot (\vec{r} \times \vec{F})$$

$$\vec{r} = 3 \hat{k} \text{ in.}$$

$$\vec{F} = -50 \text{ lb} \left\{ \cos 120^\circ \hat{i} + \cos 45^\circ \hat{j} + \cos 60^\circ \hat{k} \right\}$$

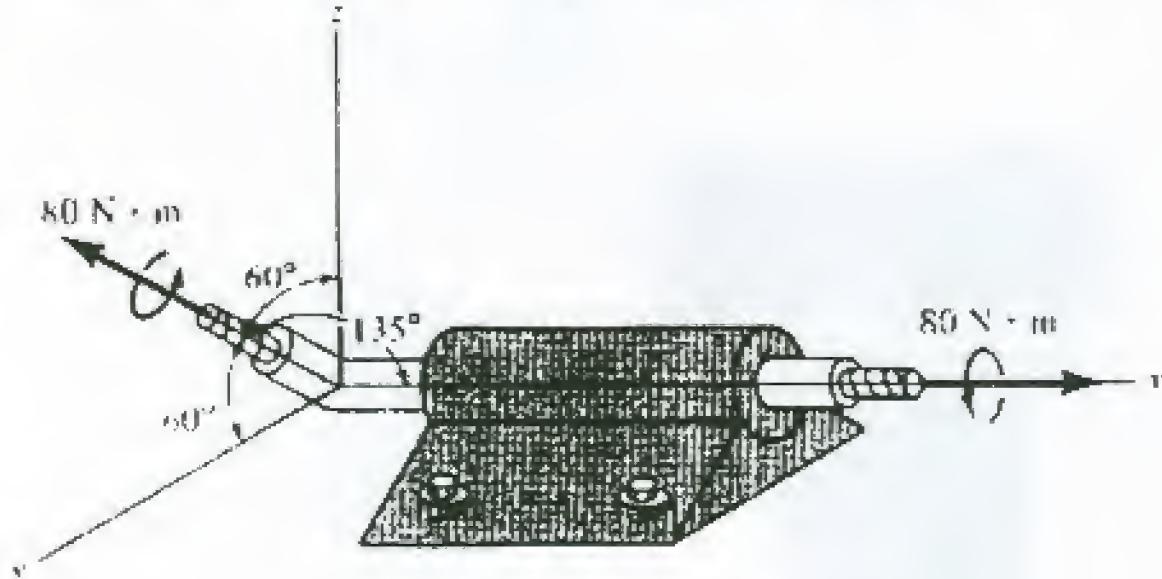
KN

$$M_y = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 3 \\ -50 \cos 120^\circ & -50 \cos 45^\circ & -50 \cos 60^\circ \end{vmatrix}$$

$$M_y = -(1)(-3)(-50 \cos 120^\circ) = +75 \text{ in. lb}$$

$$\vec{M} = M_y \hat{j} = \boxed{75 \hat{j} \text{ in. lb}}$$

4. An electrical wire cable passes through and is held firmly by the support. The cable ends are subjected to the two 80 Nm couple moments shown. The direction cosine angles are $\alpha = 60^\circ$, $\beta = 135^\circ$ and $\gamma = 60^\circ$ for the vector on the left side. Determine the resultant couple moment acting on the support specifying its magnitude and direction.



$$\vec{M} = \left\{ (80 \text{ Nm} \cos 60^\circ) \hat{i} + (80 \text{ Nm} \cos 135^\circ + 80 \text{ Nm}) \hat{j} + (80 \text{ Nm} \cos 60^\circ) \hat{k} \right\}$$

$$\vec{M} = \left\{ 40 \hat{i} + 23.4 \hat{j} + 40 \hat{k} \right\} \text{ Nm}$$

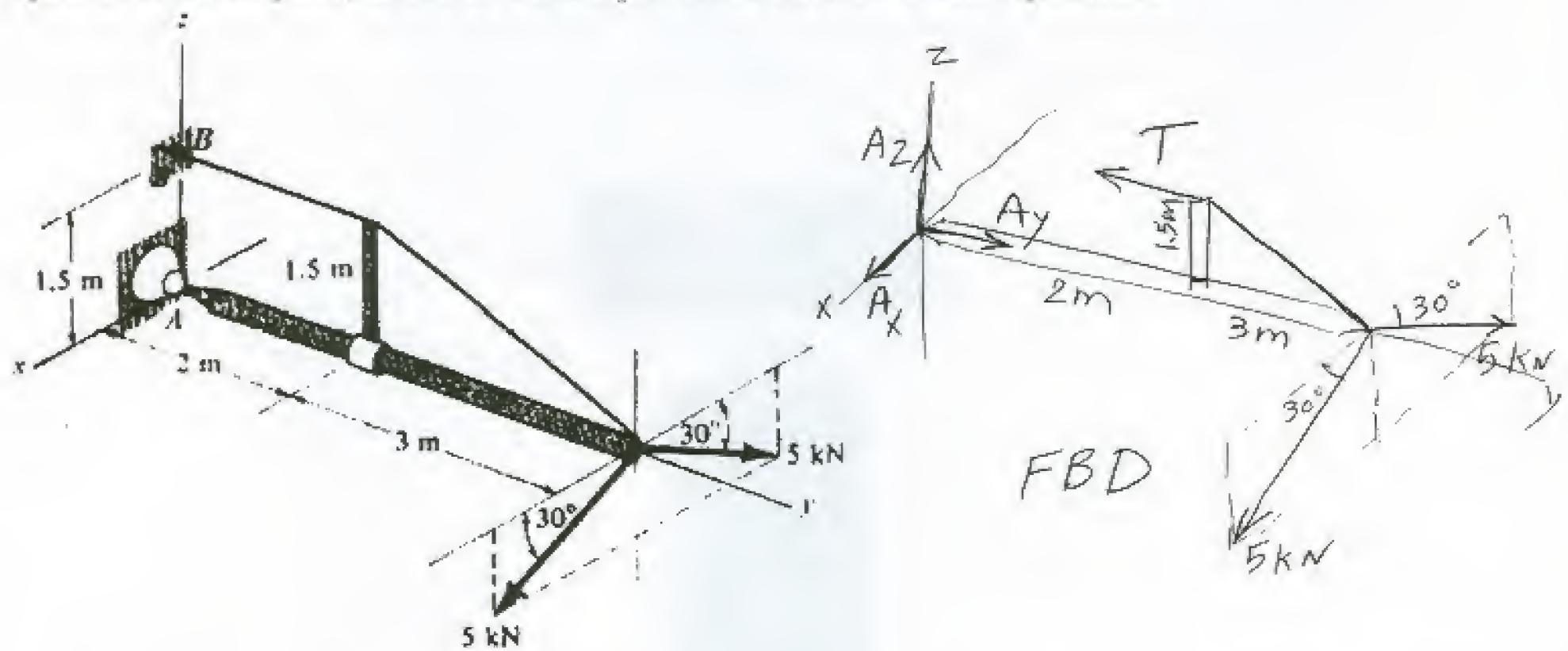
$$M = \sqrt{(40)^2 + (23.4)^2 + (40)^2} = 61.2 \text{ Nm} \Rightarrow M = 61.2 \text{ Nm}$$

$$\alpha = \cos^{-1} \left(\frac{40 \text{ Nm}}{61.2 \text{ Nm}} \right) = 49.2^\circ \Rightarrow \alpha = 49.2^\circ$$

$$\beta = \cos^{-1} \left(\frac{23.4}{61.2} \right) = 67.5^\circ \Rightarrow \beta = 67.5^\circ$$

$$\gamma = \cos^{-1} \left(\frac{40}{61.2} \right) = 49.2^\circ \Rightarrow \gamma = 49.2^\circ$$

5. The boom is supported by a ball-and-socket joint at *A* and a guy wire at *B*. If the loads in the two cables pulling on the end of the boom are each 5 kN and they lie in a plane which is parallel to the *x*-*z* plane, determine the components of reaction at *A* for equilibrium.



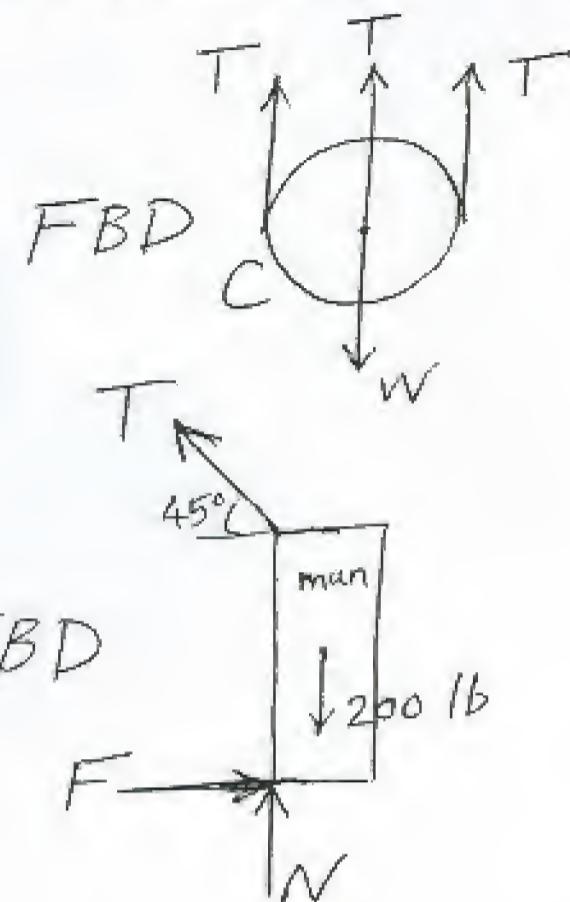
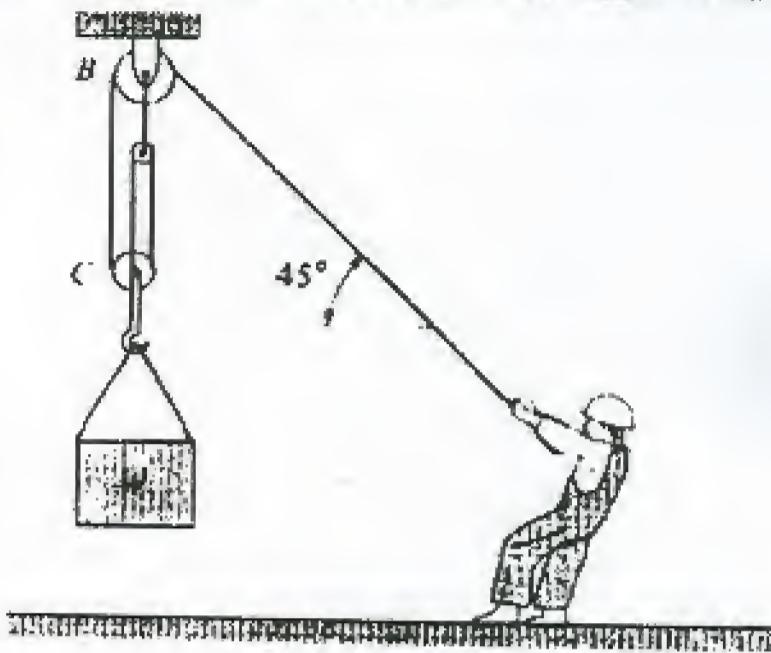
$$\sum M_x = 0 \Rightarrow T(1.5) - 2(5 \sin 30^\circ)(5) = 0 \Rightarrow T = 16.67 \text{ kN}$$

$$\sum F_x = 0 \Rightarrow A_x + 5 \cos 30^\circ - 5 \cos 30^\circ = 0 \Rightarrow A_x = 0 \text{ kN}$$

$$\sum F_y = 0 \Rightarrow A_y - T = 0 \Rightarrow A_y = 16.67 \text{ kN}$$

$$\sum F_z = 0 \Rightarrow A_z - 2(5 \sin 30^\circ) = 0 \Rightarrow A_z = 5 \text{ kN}$$

6. Determine the maximum weight W the man can lift using the pulley system. The man has a weight of 200 lb and the coefficient of friction between his feet and the ground is $\mu = 0.6$.



From FBD of pulley C: $T = \frac{1}{3}W$

From man's FBD:

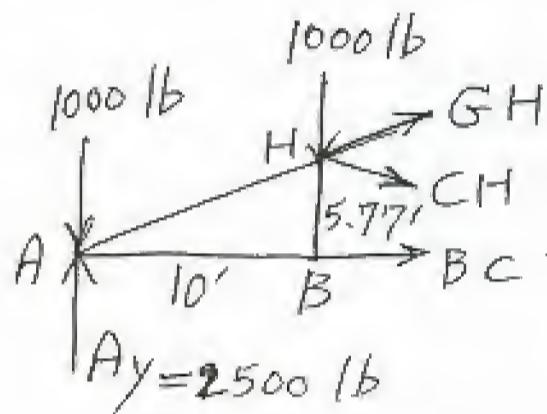
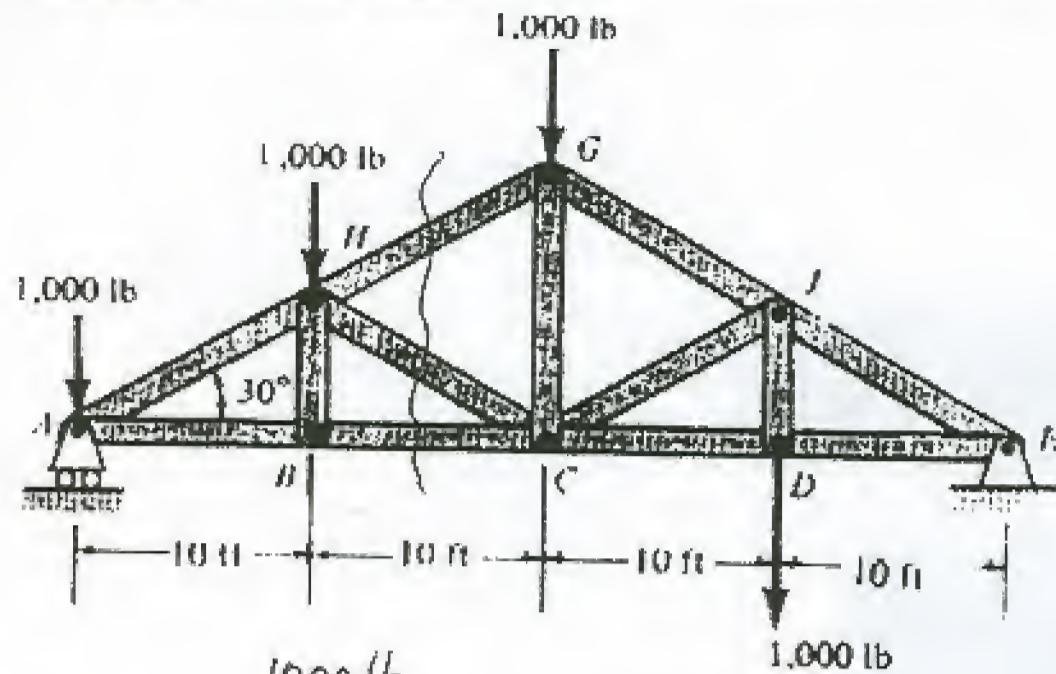
$$\sum F_x = 0 \Rightarrow -T \cos 45^\circ + F = 0 \Rightarrow F = \frac{1}{3}W \cos 45^\circ$$

$$\sum F_y = 0 \Rightarrow T \sin 45^\circ - 200 + N = 0 \Rightarrow N = 200 - \frac{1}{3}W \sin 45^\circ$$

$$F = \mu N \Rightarrow \frac{1}{3}W \cos 45^\circ = 0.6(200 - \frac{1}{3}W \sin 45^\circ)$$

$$\therefore W = 318 \text{ lb}$$

7. Determine the force in member BC of the pin connected truss. Indicate whether the member is in tension or compression.



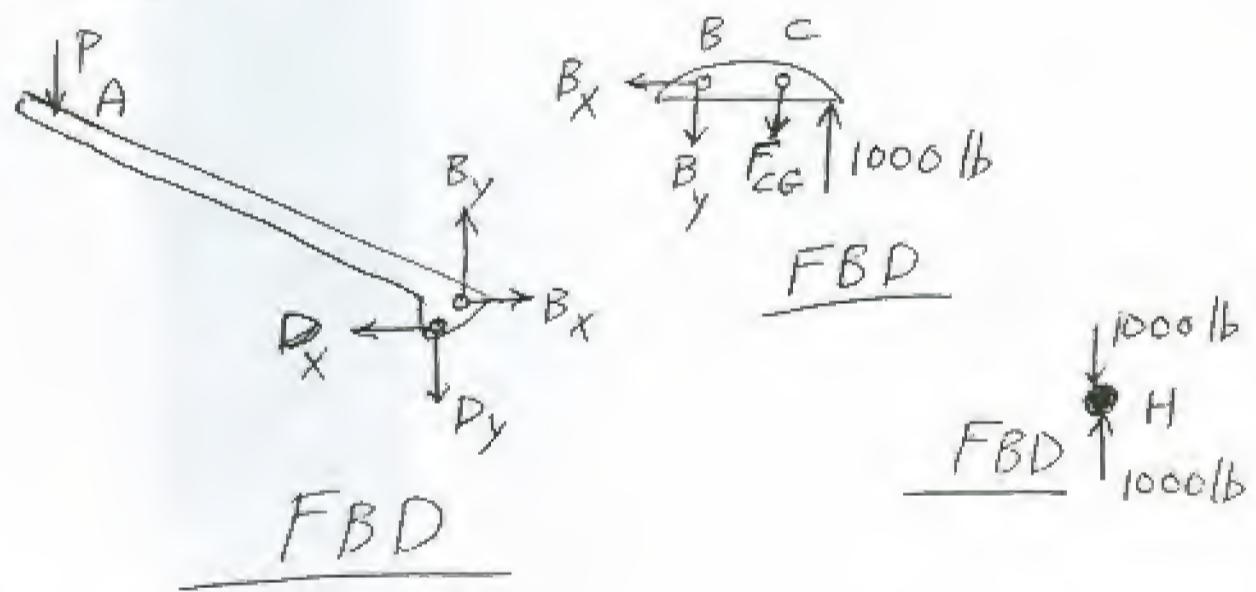
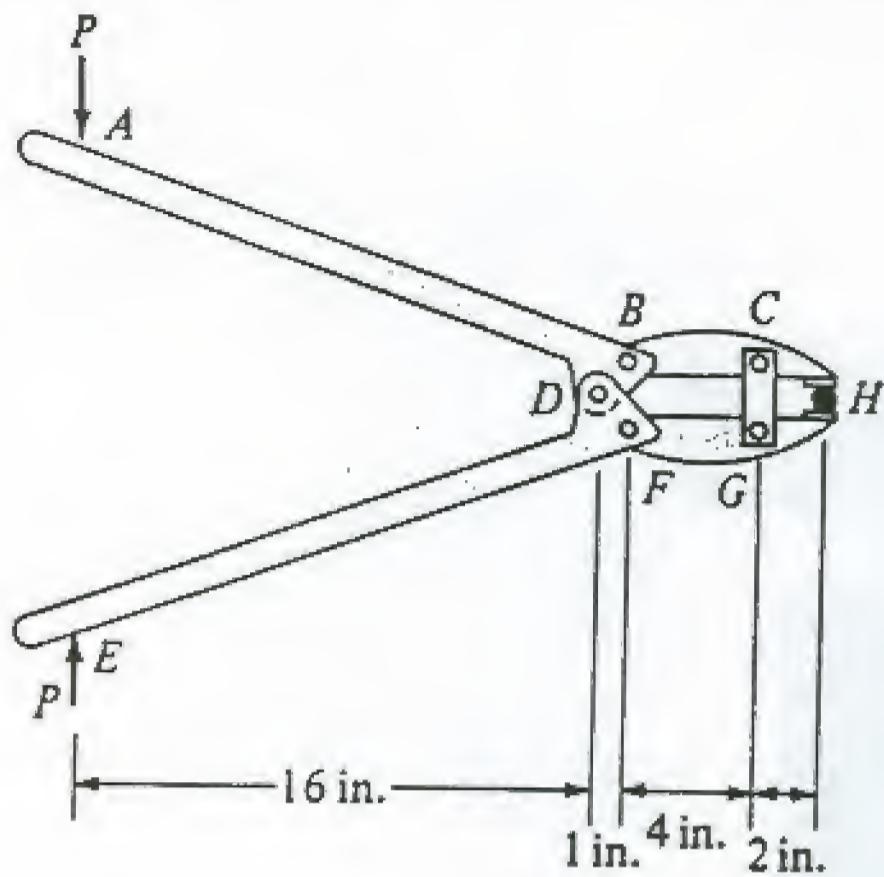
$$\begin{aligned}\sum M_E &= 0 \\ -A_y(40) + 1000(40) + 1000(30) \\ + 1000(20) + 1000(10) &= 0 \\ \Rightarrow A_y &= 2500 \text{ lb}\end{aligned}$$

$$\sum M_H = 0 \Rightarrow BC(5.77) + 1000(10) - 2500(10) = 0$$

$$\Rightarrow BC = 2600 \text{ lb}$$

$$BC = 2.60 \text{ kip}$$

8. For the bolt cutter shown, determine the magnitude of the vertical forces P that must be applied at A and E to generate vertical forces of 1000 lb on the rod, H, to be cut. Also, determine the forces on each member of the machine under this loading condition.



$$\text{Member BC: } \sum F_x = 0 \Rightarrow B_x = 0$$

$$\sum M_C = 0 \rightarrow 1000(2) + B_y(4) = 0 \Rightarrow B_y = -500 \text{ lb}$$

$$\sum F_y = 0 \Rightarrow -(-500) + 1000 - F_{CG} = 0 \Rightarrow F_{CG} = 1500 \text{ lb}$$

Member AD:

$$\sum F_x = 0 \Rightarrow D_x = 0$$

$$\sum M_D = 0 \Rightarrow P(16) + (-500)(1) = 0 \Rightarrow P = 31.25 \text{ lb}$$

$$\sum F_y = 0 \Rightarrow -31.25 - 500 - D_y = 0 \Rightarrow D_y = -531.25 \text{ lb}$$

Because of symmetry, the forces on FG and DE are

Fundamental Equations of Statics**Cartesian Vector**

$$\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}$$

Magnitude

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Direction

$$\begin{aligned} \mathbf{u}_A &= \frac{\mathbf{A}}{A} = \frac{A_x}{A} \mathbf{i} + \frac{A_y}{A} \mathbf{j} + \frac{A_z}{A} \mathbf{k} \\ &= \cos \alpha \mathbf{i} + \cos \beta \mathbf{j} + \cos \gamma \mathbf{k} \\ \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma &= 1 \end{aligned}$$

Dot Product

$$\begin{aligned} \mathbf{A} \cdot \mathbf{B} &= AB \cos \theta \\ &= A_x B_x + A_y B_y + A_z B_z \end{aligned}$$

Cross Product

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Cartesian Position Vector

$$\mathbf{r} = (x_2 - x_1) \mathbf{i} + (y_2 - y_1) \mathbf{j} + (z_2 - z_1) \mathbf{k}$$

Cartesian Force Vector

$$\mathbf{F} = F \mathbf{u} = F \left(\frac{\mathbf{r}}{r} \right)$$

Moment of a Force

$$\begin{aligned} M_O &= \mathbf{F} \cdot \mathbf{r} \\ \mathbf{M}_O &= \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \end{aligned}$$

Moment of a Force About a Specified Axis

$$M_a = \mathbf{u} \cdot \mathbf{r} \times \mathbf{F} = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Simplification of a Force and Couple System

$$\mathbf{F}_R = \sum \mathbf{F}$$

$$(\mathbf{M}_R)_O = \sum \mathbf{M}_O$$

Equilibrium**Particle**

$$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0$$

Rigid Body-Two Dimensions

$$\sum F_x = 0, \sum F_y = 0, \sum M_O = 0$$

Rigid Body-Three Dimensions

$$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0$$

$$\sum M_{x'} = 0, \sum M_{y'} = 0, \sum M_{z'} = 0$$

Friction

$$\text{Static (maximum)} \quad F_s = \mu_s N$$

$$\text{Kinetic} \quad F_k = \mu_k N$$

Center of Gravity**Particles or Discrete Parts**

$$\bar{r} = \frac{\sum r W}{\sum W}$$

Body

$$\bar{r} = \frac{\int \bar{r} dW}{\int dW}$$

Area and Mass Moments of Inertia

$$I = \int r^2 dA \quad I = \int r^2 dm$$

Parallel-Axis Theorem

$$I = \bar{I} + Ad^2 \quad I = \bar{I} + md^2$$

Radius of Gyration

$$k = \sqrt{\frac{I}{A}} \quad k = \sqrt{\frac{I}{m}}$$

Virtual Work

$$\delta U = 0$$